Population Projections of African Countries Based on Historical Rates of Fertility Decline

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Abstract

We developed a model of fertility projections for high fertility countries and applied it to produce population projections of countries of Africa from 2015 to 2100. The model is based on two basic assumptions: a) fertility transition in all African countries is now underway, and 2) fertility decline will proceed at average historical pace observed during fertility transitions in national populations. Population of Africa is projected to reach about 3 billion people by 2100, lower by 1.4 billion than the latest projections prepared by the United Nations, 4.4 billion people in 2100 (United Nations, 2015), and only 340 million higher than the projections prepared by Lutz et al. eds. (2014), 2.6 billion people. Population of sub-Saharan Africa is expected to reach 1.9 billion in 2050 and 2.6 billion in 2100. It is likely that our projections err on the high side as due to availability of modern contraceptives, ongoing international efforts directed towards expanding family planning programs, improvements in socioeconomic conditions, and continuing urbanization fertility transitions in the countries of Africa could be even faster than fertility transitions historically observed.

1. Introduction

Population projections are produced to portray future trends in population size, births, age composition, dependency ratios, spatial distributions and other demographic characteristics given a certain set of assumptions. They are used in decision making process where population trends are expected to play a major role, for planning purposes (e.g., for assessing future demand for food, water, energy, health services etc.) and for alerting the public and policymakers about major trends that may have an impact on economic development (e.g. demographic dividend) or even on national security (e.g. population decline and de-population).

In high fertility countries, most of which now in Africa, future population levels depend primarily on future levels and trends in fertility, a major driving force behind their population growth (Andreev et al., 2013). Fertility in many African countries remains high: out of 57 countries in 19 countries total fertility rate (TFR) is now about 5 or higher (United Nations, 2015). There is virtually a universal consensus within the scientific community that fertility is not going to remain at such high levels and will eventually decline. Such change-of-course expectations about future trends in high fertility countries are based on the theory of demographic transition (e.g. Caldwell, 2006): as a country moves from a pre-industrial to a post-industrial stage then both mortality and fertility decline reflecting a demographic regime. Death rates drop first and a decline in fertility follows after a delay of several decades. Excess of fertility over mortality results in a rapid population growth continuing until fertility reaches replacement level.

Figure 1 illustrates historical demographic transition in Sweden: sustained decline of infant mortality started in the early-1800s at level about IMR = 205, and decline in fertility started approximately 80 years later, close to the year 1880, at initial level about TFR = 4.5. By that time, infant mortality had declined by nearly half. Replacement level (TFR = 2.1) was reached by Swedish fertility by 1930, completing the demographic transition in this country (see also Hofsten and Lundström, 1976).

[Fig 1 about here]

This general principle of adhering to theory of demographic transition has been adopted by all major agencies and institutions producing global population projections. There is less agreement, however, on pace of fertility decline. As a result, predictions of total populations differ dramatically. In the latest 2015 revision of the World Population Prospects (WPP) prepared by the UN Population Division (UNPD) ("UN projections" hereafter) the total population of Africa is expected to reach 4.4 billion people by 2100 (United Nations, 2015). Projections produced jointly by The International Institute for Applied Systems Analysis (IIASA) and Vienna Institute of Demography (VID) (hereafter referred to as "IIASA projections") predict only 2.6 billion people in Africa by the end of this century (Lutz et al. eds., 2014).

A useful insight into future population levels and trends in Africa can be obtained by constructing a population projection variant based on average historical rates of fertility decline. Based entirely on the historical evidence with minimal assumptions about parametric representation of historical trends and incorporating no expert opinions about future, this projection scenario will provide an informative baseline against which all other projections could be compared.

The objective of this work is to produce population projections of African countries based on the historical pace of fertility decline observed during fertility transitions in national populations. In the first section, we describe the model and basic assumptions underlying it. In the next section, we discuss available empirical data, their strengths and limitations, needed to fit the model. We proceed with description of the estimation process and difficulties associated with correct selection of data for fitting the model. In the following sections, we present results of our projections for African countries and compare them with the latest projections produced by UN Population Division and by IIASA. A detail discussion of possible reasons for differences between our and UN projections is presented as well.

Model

If $TFR_{c,t}$ is fertility level in country *c* and in period *t*, and $TFR_{c,t+1}$ is fertility level in the next *t+1* projection period, 5 years later, pace of fertility decline is

$$d_{c,t} = TFR_{c,t} - TFR_{c,t+1}$$

We assume that $d_{c,t}$ depends only on the current fertility level in a country if a country had entered fertility transition. We further assume that the functional form *d()* is the same for all countries:

$$d_{c,t} = d(TFR_{c,t}) \qquad (1)$$

The model rests on assumption that the past fertility trends in a country before the fertility transition have no influence on the future fertility trends—after having entered the fertility transition stage fertility decline will proceed at a historical pace observed in the countries that have passed through their fertility transitions. The "waterfall"[1] property of this model is consistent with structural change in fertility trends predicted by the demographic transition theory.

Data

We used UN fertility estimates, which are published for 201 countries and for the period 1950-2015. Estimates of TFR are available for each 5-year period, from July, 1st year *t* to July, 1st year *t + 5*, centered at January 1st, year *t+3*. The UN estimates of trends in TFR and age-specific fertility rates, age groups from 15-19 through 45-49, are produced by applying a battery of demographic techniques. For countries with good statistical systems, fertility estimates are derived from data on vital registration and the estimates published by the National Statistical Offices (NSOs) are simply incorporated in the UN dataset. On infrequent occasions, the UN estimates are derived directly from registered births and

[1] Analogous to a river—after having entered a waterfall transition to the next river level is determined by the properties of a waterfall rather by the river flow before that.

population estimates by applying the standard demographic techniques. In this case, they might differ from the official figures[2].

For majority of the countries in the world, vital registration systems are either non-existent or incomplete rendering them unusable for producing fertility estimates. In this case, the estimates are derived from the data collected in retrospective surveys or censuses. Figure 2 illustrates estimation process for Zimbabwe. First, all empirical information on fertility levels and trends is collected and assessed for quality. Second, a smoothed curve, series WPP in Figure 2, is drawn through the empirical data points to produce preliminary estimates. As a first round of fertility, mortality and migration estimates is ready for a country, next step in the estimation process is to apply cohort component method to evaluate whether the estimated population dynamics is consistent with the available age- and sex-specific population structures from censuses and surveys. If necessary, the preliminary estimates of all demographic components are re-adjusted and fit to the age structures re-examined. The entire process is repeated until the estimates are deemed reasonable to reproduce the available empirical data well.

[Fig 2 about here]

Estimates for a particular country are produced by an analyst assigned to this country; he/she is responsible for collecting all empirical data and selecting appropriate demographic techniques. Due to the publication deadlines, it is not always possible to incorporate all available fertility data because the data from surveys and censuses has to be located and processed individually by each analyst. The data and reports are often not readily available, especially if this is a national survey not included into a global survey programs as DHS or MICS. Moreover, in addition to the fertility estimates, the UN analysts have to produce estimates of population, mortality, migration, and population projections for each of the assigned countries. The work on a revision of World Population Prospects is commonly done by a team of five people over period of six months. Each member of the team is responsible for about 45 countries. Out of 201 countries published by the UN Population Division, demographic estimates for 20 Latin American countries are produced by CELADE (Centro Latinoamericano y Caribeño de Demografia).

It is difficult to evaluate quality and empirical basis of the fertility estimates because the estimates are produced individually for each country, and only the final figures are published by UNPD. Only in 2015, UNPD undertook a first attempt to collect, organize, document and publish all available empirical fertility data used as inputs for fertility estimation (United Nations, 2015a). These data can be used for exploration of empirical basis underlying UN estimates including gaps and limitations of the final estimates. More detail review of the empirical data and the methods used to adjust the raw data will be included in the 2015 World Fertility Report (forthcoming).

[2] Calculation of fertility rates from published births and population estimates is needed, for example, if an update of population estimates have been published but the demographic rates have not been updated yet by a national statistical office.

Estimating Pace of Fertility Decline during Fertility Transition

To estimate relation $d_{c,t} = d(TFR_{c,t})$ one need to estimate onsets of fertility transition in country-specific fertility trends and select only fertility decrements after the onset of the fertility transition. We computed fertility decrements from fertility estimates published in the 2015 Revision of the World Population Prospects (United Nations, 2015), periods from 1950-55 to 2010-15, and selected only the decrements observed after the estimated onsets of fertility transitions for each country. Period 1950-2015 is an estimation period in this revision and the fertility estimates for this period are derived from empirical data. We used all data for 1950-2015, at face value, mostly on grounds of computational convenience, in a way similar to how they used by UNPD for preparing population projections. It would be important to note that there are many gaps in the empirical data, especially before 1970 and for the 2010-2015, the last estimation. This makes fertility decrements for these periods are less reliable than the rest. The European fertility transitions, mostly completed before 1950, are only partiality represented in the UN dataset as the UN series start in 1950 only.

Estimating onset of the fertility transition is a not a trivial task as it is not a well-defined concept. Consequently, there is no a well-established procedure for producing an estimate. In the Princeton project the period with a 10 percent of decline in fertility was regarded as a period when it could be safe to assume that fertility transition is underway (Coale and Treadway, 1986). Casterline (2001) argued that “a more defensible approach is to regard the peak from which fertility declines as the starting point for the decline”. Bongaarts (2002) further suggested reducing the threshold to 5 percent to address certain criticism of the past practice. Alkema et al. (2011) , however, working also with UN fertility estimates, found that the threshold of 5% is “too stringent” for countries in which fertility transition started off slowly, e.g. Mozambique, and further proposed to select “for the start period of the fertility transition as the most recent period with a local maximum within 0.5 child of the global maximum”. If the global maximum is below 5.5 the Alkema et al.’s method assumes that the earlier stages of the fertility transition are not observed and the onset is somewhere before 1950, the starting year of the UN dataset.

As a part of work on the 2015 World Fertility Report (forthcoming), UNPD has produced estimates of onsets and duration of fertility transitions for all 201 countries by all three methods. The estimates are based on a preliminary version of an expanded dataset, which includes all available reliable historical estimates for the pre-1950 periods by incorporating all historical estimates. Figures 1 and 3 illustrate difficulties arising on application of each of three methods. The most reliable and extended series of mortality and fertility indicators are available for Sweden (Fig. 1). Swedish estimates are based on data from vital registration and available for the period from 1750 to 2013. In Sweden, the global TFR maximum, close to 5, is observed for the first period 1751-1755 (yellow circle). The methods based on the 5% and 10% fertility declines set then onset of fertility transition to 1757 (cyan circle) and 1770 (green circle), respectively. No estimate is produced by Alkema et al.’s method as the maximum fertility level is less than 5.5. One must assume on applying this method that onset of fertility decline is before 1750. Obviously, all three methods fail to estimate correctly onset of fertility transition in Sweden. The transition in Sweden started around 1880 (red circle) and had been completed by 1930 (red square) (Hofsten and Lundström, 1976). Over this period fertility declined from a long-term level of about 4.5 to 2 on average pace of 0.5 children per decade.

Figure 3 shows an example of estimation of timing of a contemporary fertility transition. The estimation period, 1950-2015, has a single global maximum, TFR = 7.4, year 1968, and Alkema et al.'s method selects this year and level of fertility as onset of the fertility transition in Angola (there are no local maximums). Due to very flat trends, the 5% method places onset at 1997, and the 10% method at 2007, almost 40 years later as compared with Alkema's estimate. Currently available empirical data for Angola start in the earlier 1980s[3]. Only in the late 2000s there are some weak signs of fertility decline as supported by the available empirical evidence. For the years before 1980 we do not really have any empirical evidence on levels and trends in fertility in Angola. Placing onset of fertility transition at 1968 would be a mistake. A better estimate, still dubious, is to use the 10% rule and select 2005-10 as a first period of fertility decline. This implies that before 2007 Angola was still in a pre-transitional stage.

[Figure 3 about here]

These two illustrative examples show that a single mechanical rule cannot be used under all circumstances. For this reason, we selected estimates of fertility transitions by through country-by-country examinations using the estimates produced by the above methods as guidance only. Figure 4 shows both the data *selected* for estimation of $d_{c,t}$, series dTFR (hollow pale blue circles) and the data *excluded* from the estimation, series "dTFR (excluded)" (grey crosses), the empirical fertility decrements in that refer to the periods with pre-transitional fertility,.

We decided not impose any particular parametric form on this function[4]. Instead, we assumed that $d(TFR_{c,t})$ could be well approximated by a spline. Selection of a particular spline, degree and smoothness, is driven entirely by the observed empirical data. The $d(TFR_{c,t})$ was estimated non-parametrically by fitting several spline functions to the empirical data (Fig. 4). The first estimate, labelled "Histogram", simply computes averages over TFR levels 1-2, 2-3, etc. This simple estimate, a zero degree spline, has an advantage that it depends only on the data points in the selected fertility interval and it does not depend on the data points outside of the interval. The estimates labelled "Spline …" show fit of a cubic spline with varying smoothing parameter: the closer smoothing parameter is to zero the smoother is the spline, and, the closer is the smoothing parameter to one, the less smoothing is introduced in the estimates. The estimate labeled "spline (p=GCV)", is a cubic spline with smoothing parameter selected automatically by minimizing the general cross-validation statistics.

All estimation procedures returned remarkably similar and simple results. If fertility is above 4.5, fertility declines approximately at a pace 0.5 children per woman per quinquennium. This pace of decline is virtually constant for any fertility level up to TFR = 7.5 (recall also that 0.5 it was average pace off decline during fertility transition in Sweden). The splines with higher degree of smoothing (series

[3] Purple diamonds and dashed series in Fig. 3. See also
http://www.un.org/en/development/desa/population/publications/dataset/fertility/wfd2015/index.htm

[4] In the earlier models UN Population Division used a double logistic curve for modelling fertility declines during fertility transitions for high fertility countries (United Nations, 2006).

“spline (p=0.05)” and “spline (p=GCV)”) produce a slight increase in the estimated pace of decline but it is unlikely to be statistically significant due to high variation in the data. As fertility falls below 4.5, fertility decline decelerates and pace of fertility decline decreases virtually linearly reaching zero at a level of about 1.5 children per women. Below TFR=1.5, the pace of decline becomes negative indicating that fertility will actually increase once it drops below this level.

The level of fertility approximately equal to 1.5 is a point of a stable equilibrium, the level to which all fertility trajectories will converge in a long run[5]. For low fertility countries, this equilibrium could be used to project the convergent levels of total fertility, which have been arbitrary assumptions so far in the UN projections. Furthermore, this equilibrium is quantitatively close to the estimated values of the economically optimal equilibrium recently proposed by Lee and Mason and their colleagues (2014). This closeness calls for further explorations.

Due to high variation in the data and uncertainty about fertility estimates we decide to use the GCV spline for the projection proposes. In addition, general cross-validation technique is well-explored and widely accepted statistical technique for selecting a smoothing parameter. The shape of the GCV spline can be further reduced to a linear spline well captured by a straight line for TFR < 4.4 and a constant line 4.4, or by 3 parameters only. As effect on population projections is expected to be minimal we have not attempted further simplifications of the functional form of $d(TFR_{c,t})$.

Projections and Comparisons with UN and IIASA projections for Africa

We computed projections of future trajectories of fertility for all African countries by applying estimated $d_{c,t} \sim spline(TFR_{c,t})$ to the current level of fertility in the period 2010-15:

$$TFR_{c,t+1} = TFR_{c,t} - d_{c,t}$$

We assumed that all African countries have started now their fertility transition. The projected fertility trajectories have been used as input to the cohort component method for producing population projections for the period 2015-2100. Our assumptions about future trends in mortality, migration and age patterns of fertility are consistent with the assumptions underlying the 2015 Revision of World Population Prospects (United Nations, 2015). The projections have been run separately for each of the African countries[6] and the projection for the entire continent is sum of individual projections.

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[5] This was suggested by Nan Li. It can be formally proved by applying theory of stability of ordinary differential equations originally developed by Lyapunov (1892). Heuristically, once TFR falls below 1.5 the next decrement will be negative and TFR will continue to increase until it reaches a level above 1.5. After this time, TFR will decline and the entire process will repeat itself. In a long run, the system will converge to the TFR level of 1.5 except for a single special case when the decrements will be *exactly* offset by the increments and the system will be in a bifurcation mode jumping from one fertility level to another.

[6] Africa comprises 58 not 57 countries. We did not produce projections for St. Helena as its population is very small. We did include it in the total population of Africa.

Figure 5 compares results of our projections for Africa, series "Historical Pace of Fertility Decline" with the UN projections, series "WPP Medium", and IIASA, series "SSP2 (IIASA)" . All three projections predict that population growth of African population will continue at least through the end of the century. The strongest and sustained growth is expected in the UN projections: the total population of Africa is expected to reach about 2.5 billion in 2050, and continue to increase close to 4.5 billion in 2100. Our "Historical Pace of Fertility Decline" projection is characterized by intermediate growth rates: population is expected to reach about 2.3 billion people in 2050 and about 3 billion people in 2100. The slowest growth is found in the IIASSA projection: population is reaching slightly above 2 billion in 2050 and about 2.6 billion in 2100. By the end of the century our projection is much closer to that of IIASA predicting only 340 million, or about 10 percent more than that of IIASA. Both projections, however, strikingly lower than the UN projections by 2100: our projections lower by about 1.4 billion people and the IIASSA projection are lower by about 1.8 billion people. Somewhat surprisingly and unexpectedly our projection came up quite close to the low variant of the UN projections. This suggests that fertility projections in the UN medium variant are on average 0.5 child higher than fertility projections produced by adopting historical pace of fertility decline.

In the 1950s annual population growth rate in Africa was about 2% and on a rise (Fig. 6). It reached a peak of about 2.8% in the early 1980s and dropped down to about 2.5% by the late 1990s. After that time the growth rate was staying approximately constant at 2.5% level. In all projections future growth rates are predicted to decline virtually linearly through the end of the century. In the "Historical Pace of Fertility Decline" and IIASSA projections the growth rate drops nearly to zero by 2100. In the UN projections, however, high population growth rates are expected to persist over the entire projection period. The growth rates are still high in the second half of the 21st century: about 1.7% and in 2050 and 0.7% in 2100. In historical perspective, such high growth rates over an extended period are anomalous. In Sweden, for example, average peak growth rate over the demographic transition period, from 1820 to 1875, the period when mortality decline was underway while fertility decline had not started yet (Fig. 1), was only about 1%.

[Fig. 6 about here]

Faster fertility declines lead not only to lower population sizes and to lower population growth but to lower dependency ratios as well. Excess of working age population can lead to more rapid economic gains realized through the first demographic dividend (Lee and Mason, 2006). Figure 7 compares ratio of working age population (ages 20 to 65) to combined number of children and older population (0-19 and 65+). For the entire projection period our and IIASA projections predict higher shares of working-age population as compared with UN projections. Both series shows an accelerated growth peaking in 2075 and 151 and 143 levels, respectively. UN series grows at more moderate pace levelling off at 130 by 2100.

[Figure 7 about here]

Projections and Comparisons for Individual Countries

Our and UN projections of TFR are contrasted with help of diagrams plotting fertility decrements (dTFR) vs. total fertility (TFR) for each estimation and projection period. An example of such diagram for Uganda[8] is given in Figure 8. TFR level in 1950-54, the start of the estimation period, is marked by a large yellow circle. As the yellow circle lies below zero, fertility decrement from 1950-54 to 1955-59 period was negative for Uganda meaning that fertility actually increased (slightly) over this quinquennium. Over time, as indicated by the arrow, this increase in fertility moves the yellow circle to the right, the first green circle. This green circle corresponds to the second estimation period, 1955-59. Fertility continues to move along the green circles until it reaches the pale red circle, the last estimation period, 2010-15. As indicated by positive decrements for the last estimation periods fertility was declining after an initial increase. The pale red circle marks an onset of fertility projections: UN projections are depicted by the cyan squares and our projections by the red square markers. By design our projection follow the estimated average pace of decline, the red curve. The UN projections are produced by the Bayesian hierarchal model (United Nations, 2015b and Alkema et al., 2011). Obviously, as follows from Figure 8, UN fertility decrements are consistently below the average historical decrements for the entire projection period. This leads to slower projected fertility declines and to higher future population sizes.

[Figure 8 about here]

Pooled data on pace of fertility declines (Fig. 9) shows that this pattern of difference between two projections is typical for nearly all African countries: UN fertility decrements (blue squares) are consistently below the decrements estimated by our model (red squares). For TFR > 3 the UN fertility decrements are on average 30% lower than historical values and for TFR between 2.1 and 3 the relative difference is about 40%. Only for three countries, Ethiopia, Libya and Algeria, UN declines of TFR are initially faster than in our variant. Over time, however, the rate of decline slows down, and the fertility decrements for these three countries fall below the average historical levels as well, as for other countries. Comparison with “Fast/Fast” and “Slow/Slow” models of the United Nations (2006) [9], the models used by UN Populating Division for projection fertility in high fertility countries in the earlier revisions of World Population Prospects reveals a similar pattern: all projected fertility decrements for individual countries are below the decrements implied by these models. Further comparison with the estimate of the “world average” (Alkema et al., 2011) produces similar results: all individual country-specific data points are below the “world average”.

[Figure 9 is about here]

[8] Diagrams for other countries are included in the web appendix figure: http://kirillandreev.com/afrproj/fig01/. Fertility estimates from the 2015 WPP Revision are available at http://www.un.org/en/development/desa/population/publications/dataset/fertility/wfd2015/index.htm.

[9] “Fast/Fast” stands for fast fertility decline, and “Slow/Slow” stands for slow fertility. The fertility decrements implied by “Fast/Fast” and “Slow/Slow” models are not entirely comparable with the current models as they are based on fertility estimates available before the 2004 Revision of the World Population Prospects. Since the 2004 revision empirical data for more than 10 years became available.

The analysis clearly demonstrates that the current UN model produces slower fertility declines for individual African countries as compared with 1) the declines observed earlier in the countries that have passed through their fertility transitions, 2) the declines implied by the models used before by the UN Population Division and 3) the declines of the "world average" of the Bayesian hierarchal model itself. Fertility declines for individual countries in the current UN model "can be viewed as weighted averages of a "world pattern" and information from the country data" (Alkema et al., 2011). Obviously, the UN model puts much more weight on the historical trends in the individual African countries rather than on the data on fertility transitions worldwide. The history of demographic trends in African countries appears to affect projections for the entire projection period, from 2015 through 2100. As demographic transition is a structural change in a demographic regime, the pre-transitional levels of mortality and fertility are not expected to carry crucial and long-lasting information about post-transitional levels and about duration of a fertility transition. Overly relying on the historical trends in African countries, with the estimates mostly for pre-transitional periods and often of dubious quality, for modelling future fertility transitions does not seem as the best way for projecting fertility for these countries.

One of the reasons why historical trends in individual African countries significantly affect projected fertility while "world average" plays a minor role might be an excessive number of parameters of the UN model. The UN model uses double logistic curves (United Nations, 2006) to model fertility decrements both for individual countries and for the "world average". Each double logistic curve needs six parameters to be estimated. Alkema et al. (2011) uses a special 5-parameter case of the double logistic curve with constraint *k1* = *k2* but, instead, introduces an additional gamma parameter. The total number of parameters is still six. In all, for 201 countries, there are 1212 = 6 * (201 + 1) parameters of the double logistic curves to be estimated. By adding 15 hypoparameters the total number of parameters needed to fit the BHM model is at least[10] 1227. The UN dataset includes TFR estimates for 5-year periods for 201 countries, or only 2502 = ((2015-1950)/5-1)*210 empirical data points on fertility decrements (this number includes both data for pre-transitional and post-transitional fertility). On average, an estimate of a single parameter is informed by two empirical data points only. Excluding pre-transitional and post-transitional fertility decrements would further significantly decrease this number of empirical data points per parameter. It appears that UN dataset may not have enough empirical observations to fit the BHM model due to a large number of parameters that needs to be estimated. It is certainly true for the African countries, the countries that just started their fertility transitions: a handful of empirical data points with positive fertility decrements must inform the entire shape of a double logistic curve for modeling fertility decline from the pre-transitional through the replacement level. In Figure 8, for example, the entire shape of the individual double logistic curve must be informed by empirical points between the yellow and the pale red circles only. The expectation behind the UN model is that in the case of insufficient empirical data for estimating individual double logistic curves, the decline in a country will follow the "world average". Figure 9 suggests that it is not the case.

[10] One needs also to include here parameters for prior distributions, which we did not count.

Discussion

We have developed a new model of fertility decline during a fertility transition. The model produces a projection variant based on average pace of fertility decline observed during fertility transitions in national populations. On fitting to the data on transitional fertility decrements, the model turned out to be a remarkably simple: a straight line below TFR = 4.4 and nearly constant above this level. It could be well approximated by three parameters only. In this model history of fertility trends in African countries is expected have no effect on a fertility transition. Consistent with the theory of demographic transition, we have assumed that fertility transition is a structural change in fertility trends and fertility decline depends only on the present level only. We have further assumed that pace of fertility decline will be the same for all African countries. This assumption is obviously overly simplistic but it still produces informative projections for individual countries: what would happen if fertility decline in a country will proceed at an average historical pace?

The model also produces an estimate of fertility, about 1.5, to which all fertility trajectories will converge in a long run, the level close to the estimated values of the economically optimal equilibrium (Lee and Mason, 2014). Further work is needed to provide an explanation for phenomenon.

Application of the model to project of population of African countries have produced about 3 billion people by 2100, a number lower by 1.4 billion than the latest projections prepared by the United Nations, 4.4 billion people in 2100 (United Nations, 2015). On the other hand, the model produced population that is only 340 million higher than the projections prepared by Lutz et al. eds. (2014), 2.6 billion people. How plausible is the number of 3 billion by 2100? In our view, our projections are likely to err on the high side. Due to availability of modern contraceptives, ongoing international efforts directed towards expanding family planning programs, improvements in socioeconomic conditions, and continuing urbanization fertility transitions in African countries could proceed at even faster pace than historically observed. In several African countries, the recorded fertility declines have already exceeded the average historical levels e.g. Algeria, Libya or Rwanda, Botswana, Swaziland and South Africa in sub-Saharan Africa.

Detail comparisons with the latest UN projections (United Nations, 2015) reveals that on switching to the Bayesian hierarchal model for projecting fertility in the 2010 Revision of World Population Prospects UN Population Division produces now slower fertility declines for African countries by historical standards. This deceleration of fertility declines is likely influenced by incorporating more information from the country-specific trends and less from the worldwide experience by the current model than before into fertility projections. Faster fertility declines predicted by our projections will lead to slower population growth and lower dependency ratios. Both factors will re-inforce socio-economic development via higher demographic dividends. It must be recognized though, that the concept of a demographic dividend, an accelerated economic growth due to higher than average proportion of the population of working-age, implies that the right policies are in place for it to be realized. If fertility to decline faster, accelerated work on the part of the policy makers will be required to put the right policies in place.

"All models are wrong, but some are useful". This citation from Roger Box has been often verbally stated by James Vaupel, Founding Director of the Max Planck Institute for Demographic Research, Rostock. The model presented here might be simplistic and wrong but we believe that it is certainly useful. A prevalent syndrome of beliefs is that sub-Saharan Africa is exceptional in their demographic transition and fertility declines will be slower than in other parts of the world. This view is in part supported by the recent fertility stalls in a number of sub-Saharan countries. This belief is also indirectly supported by the current UN projections. The future, however, is not necessarily governed by the past. The model presented here provides useful insights into population dynamics of the African continent if historical trends to be followed. It still likely communicates a conservative outlook—there is a great potential for fertility declines to be faster and for social-economic benefits to be higher.

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Web Appendix

Auxiliary material for this article is available online at http://kirillandreev.com/afrproj/ :

- Diagrams of decrements of TFR vs. TFR for all estimation and projections periods: http://kirillandreev.com/afrproj/fig01/;
- Projected population by age and sex for all countries: http://kirillandreev.com/afrproj/population-by-age-and-sex.xlsx.

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Figure 1 Historical Fertility and Mortality Trends in Sweden

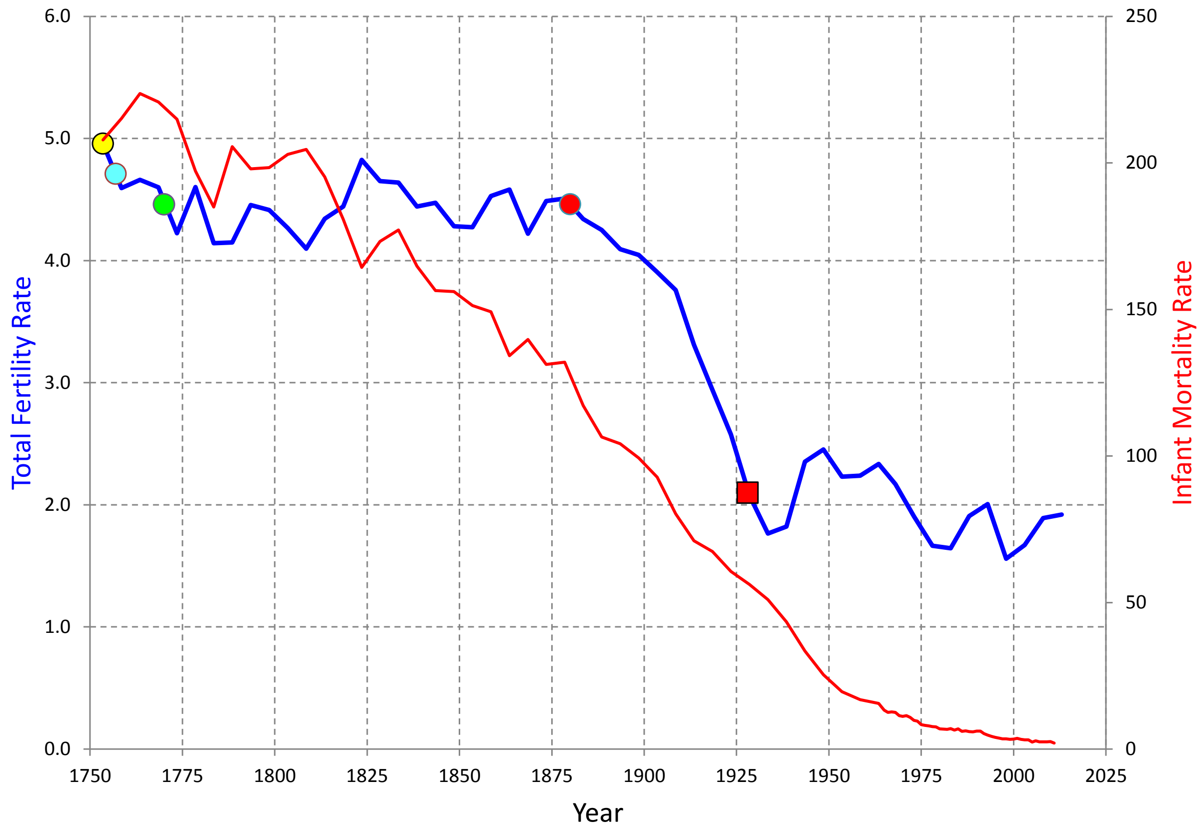
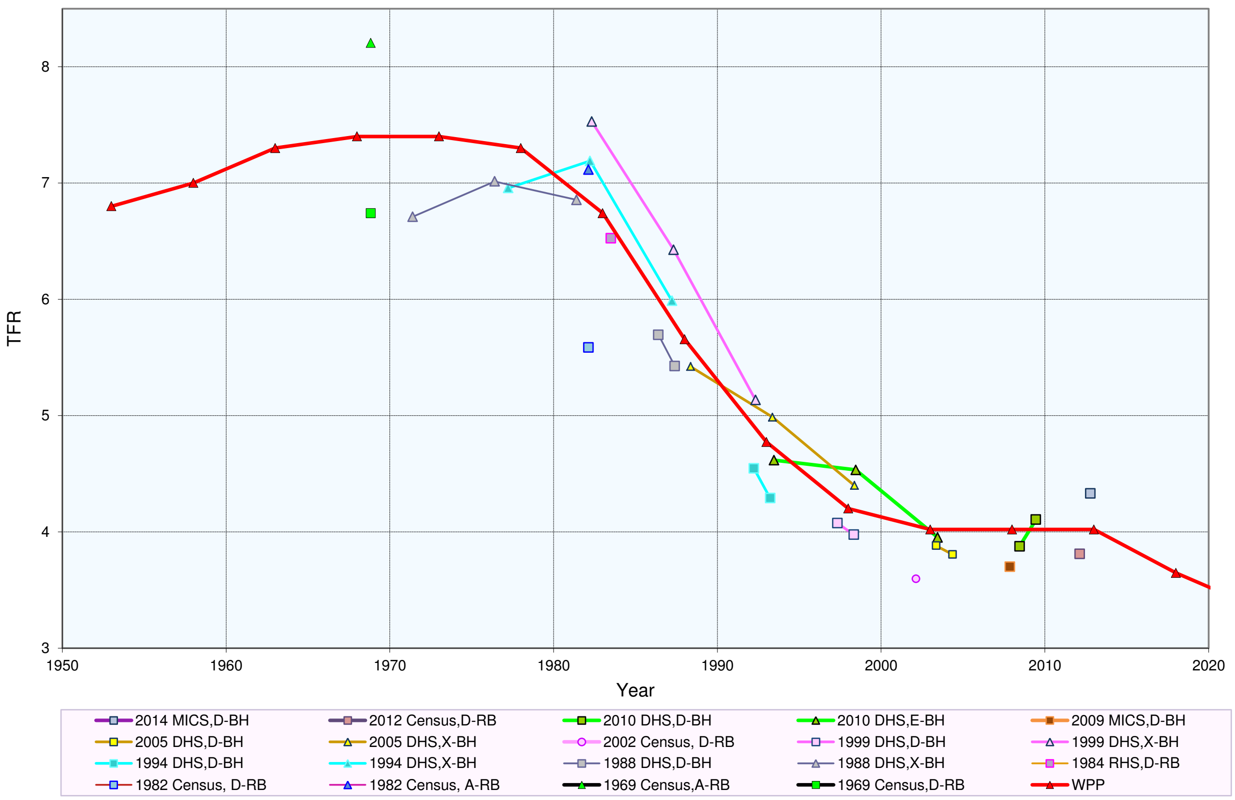


Figure 2 Fertility Trends in Zimbabwe, Estimates and Empirical data



Legend:

MICS - Multiple Indicator Cluster Survey;

DHS – Demographic and Health Project;

RHS – Reproductive Health Survey;

D – direct estimates, occurrence / exposure ratios;

X – extrapolated estimates. Extrapolations are used for deriving fertility trends from birth histories. For the lower age groups fertility rates are computed directly, and for older ages groups fertility rates are obtained by extrapolation by using data both from the lower age groups and from the previous periods. The level of fertility, TFR, is based then partially on direct and partly on extrapolated birth rates;

A – adjusted estimates (e.g. Brass P/F ratio method, Arriaga's method etc.);

BH – birth histories;

RB – recent births in the household, commonly over last 12 months;

WPP – UN estimates published in the 2015 Revision of the World Population Prospects (United Nations, 2015).

Figure 3 Angola, Estimates of Onset of Fertility Decline

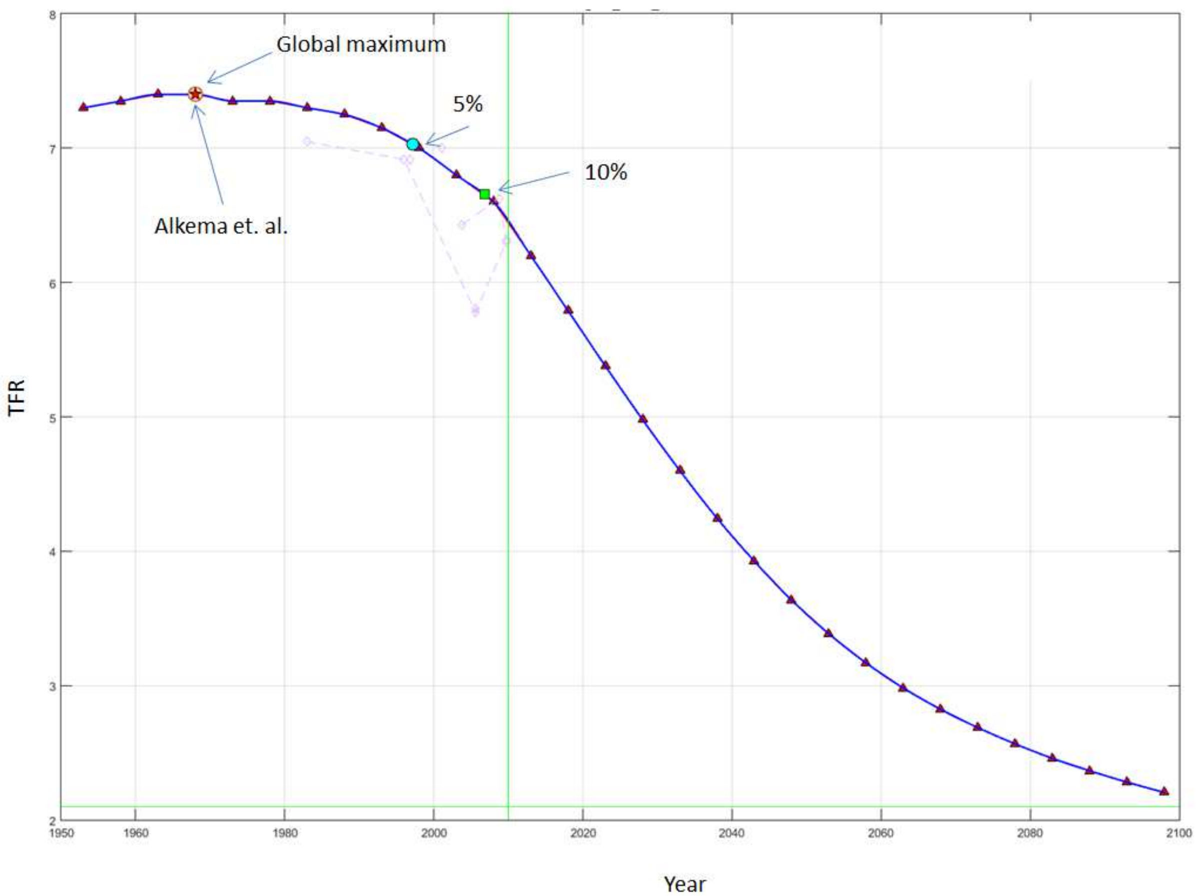
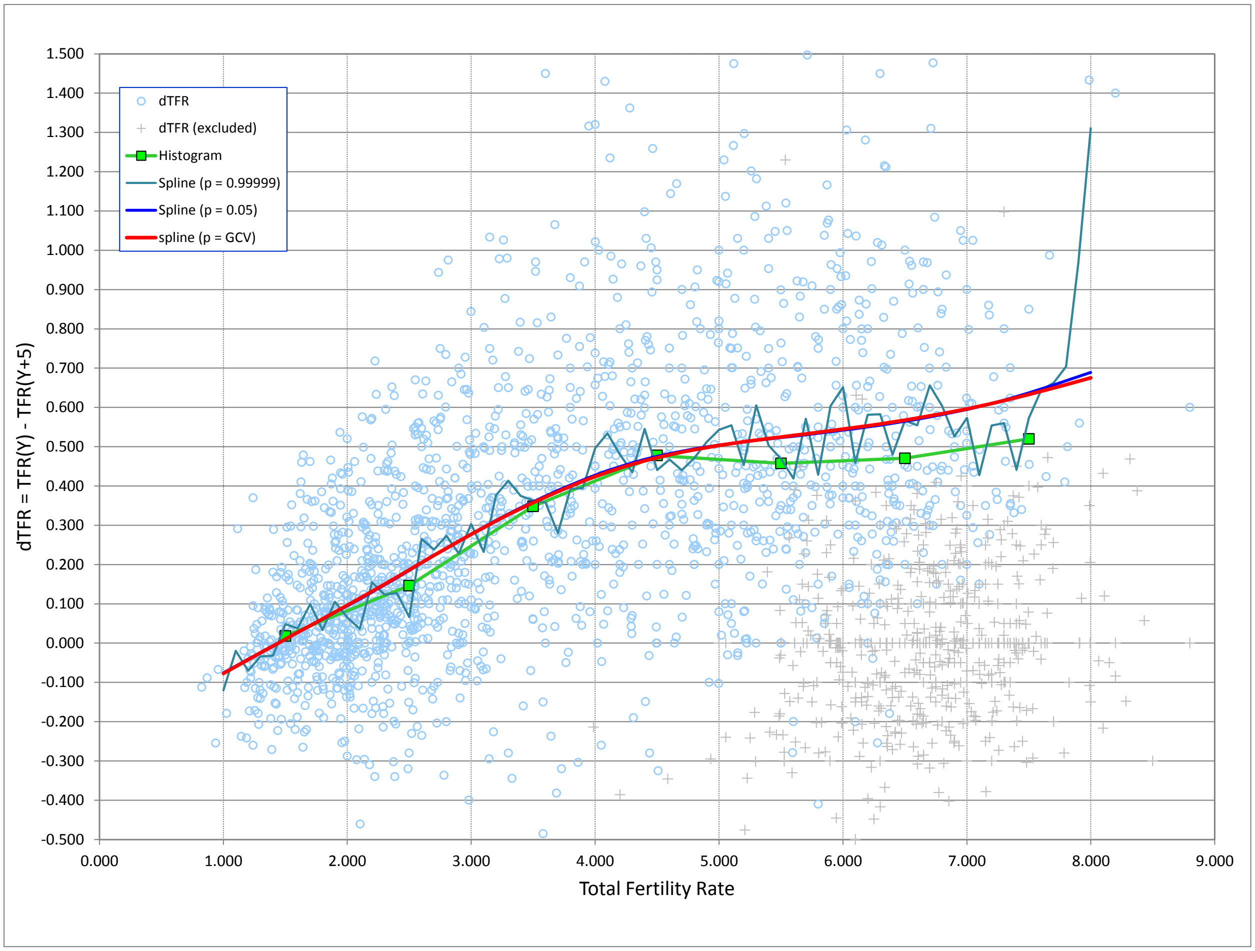
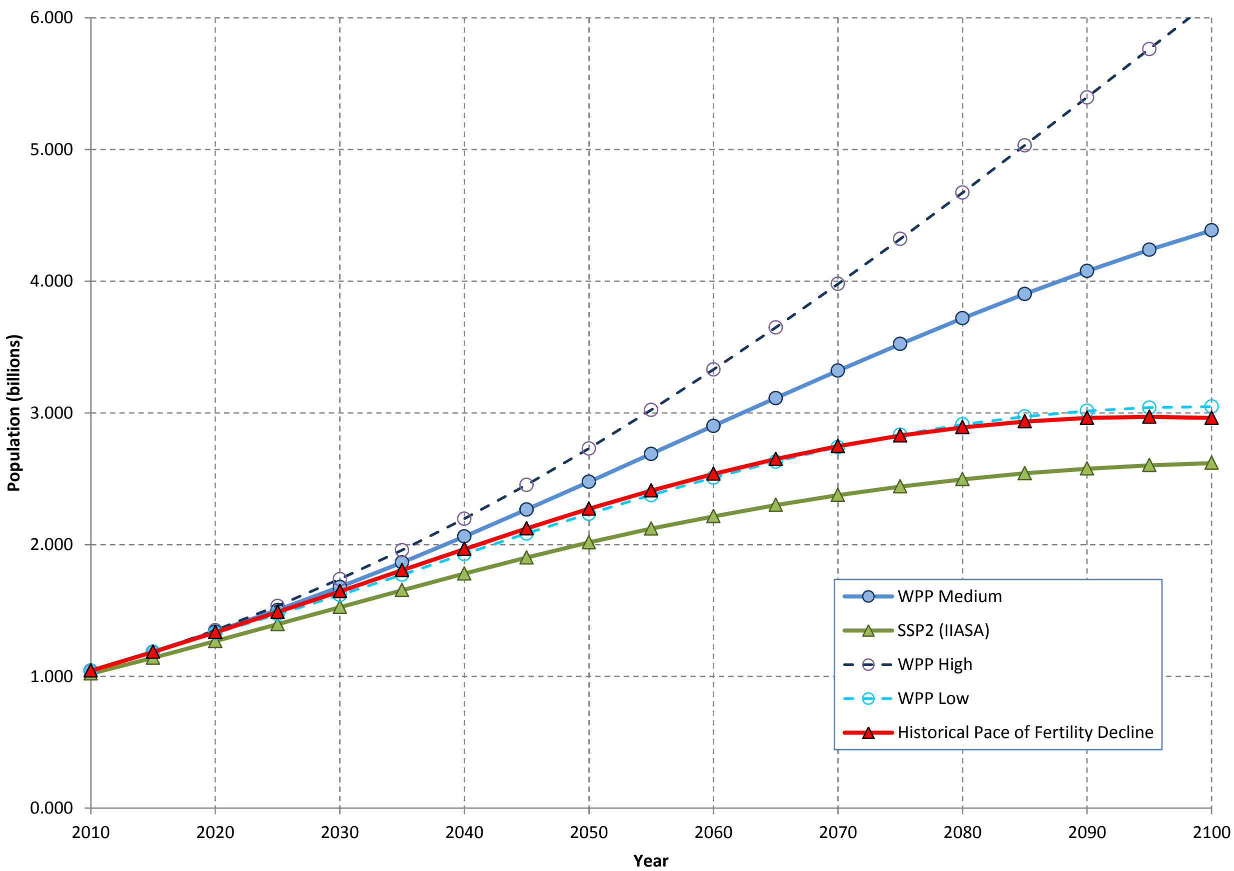


Figure 4 Estimation of Pace of Fertility Decline during Fertility Transition Period



“dTFR” – pace of fertility decline, difference between fertility level in the current period and in the period 5 years later.
“dTFR (excluded)” – as the “dTFR” series but only for the countries and periods before the onset of the fertility transition.
“dTFR (excluded)” empirical data points are not used for estimation.
“Historgram” – mean values of “dTFR” over a range of TFR levels.
“Spline (p=0.99999)” – cubic smoothing spline with smoothing parameter equal 0.99999 (small degree of smoothing).
“Spline (p=0.05)” – cubic smoothing spline with smoothing parameter equal 0.05 (high degree of smoothing).
“Spline (p=GCV)” – cubic smoothing spline with smoothing parameter automatically selected by minimizing the general crossvalidation statistics.

Figure 5 Population Projections of Africa by Three Scenarios (in billions)



"Historical Pace of Fertility Decline" – projections produced by our method based on historical paces of fertility decline during fertility transitions.
"WPP Medium", "WPP High", "WPP Low" – UN projections, the 2015 Revision of World Population Prospects (United Nations, 2015), medium, high and lower variants, respectively.
"SSP2 (IIASA)" – projections produced jointly by The International Institute for Applied Systems Analysis and Vienna Institute of Demography (Lutz et al. eds., 2014) (available online via Wittgenstein Centre Data Explorer (http://www.oeaw.ac.at/vid/dataexplorer/). The SSP2 scenario is described as "the middle of the road scenario that can also be seen as the most likely path for each country".

Selected series values:

Year	Fertility Transition	2015 WPP	SSP2 (IIASA)
2010	1.044	1.044	1.022
2030	1.646	1.679	1.526
2050	2.273	2.478	2.017
2075	2.828	3.525	2.442
2100	2.962	4.387	2.620

Figure 6 Annual Population Growth Rates in Africa, Estimates and Three Projection Scenarios (percent)

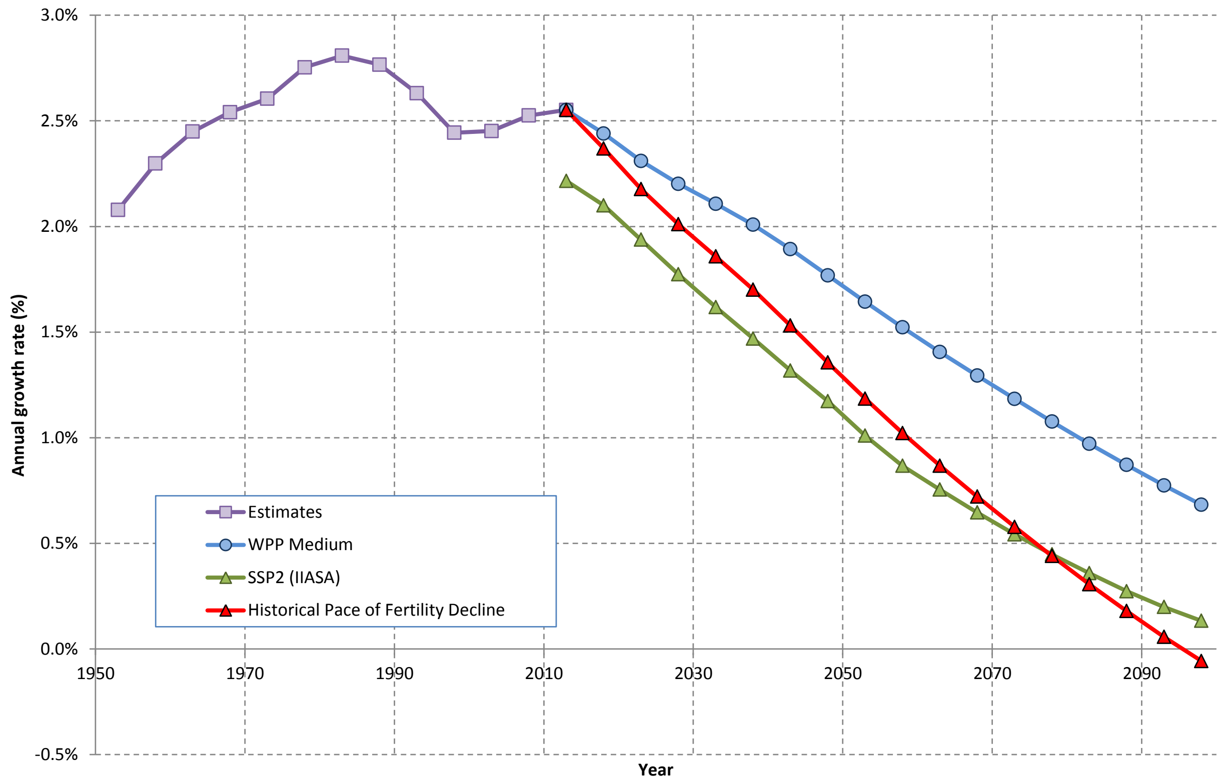


Figure 7 Ratio of Population at Age Group 20-64 to Population at Age Groups 0-19 and 65+ in Africa

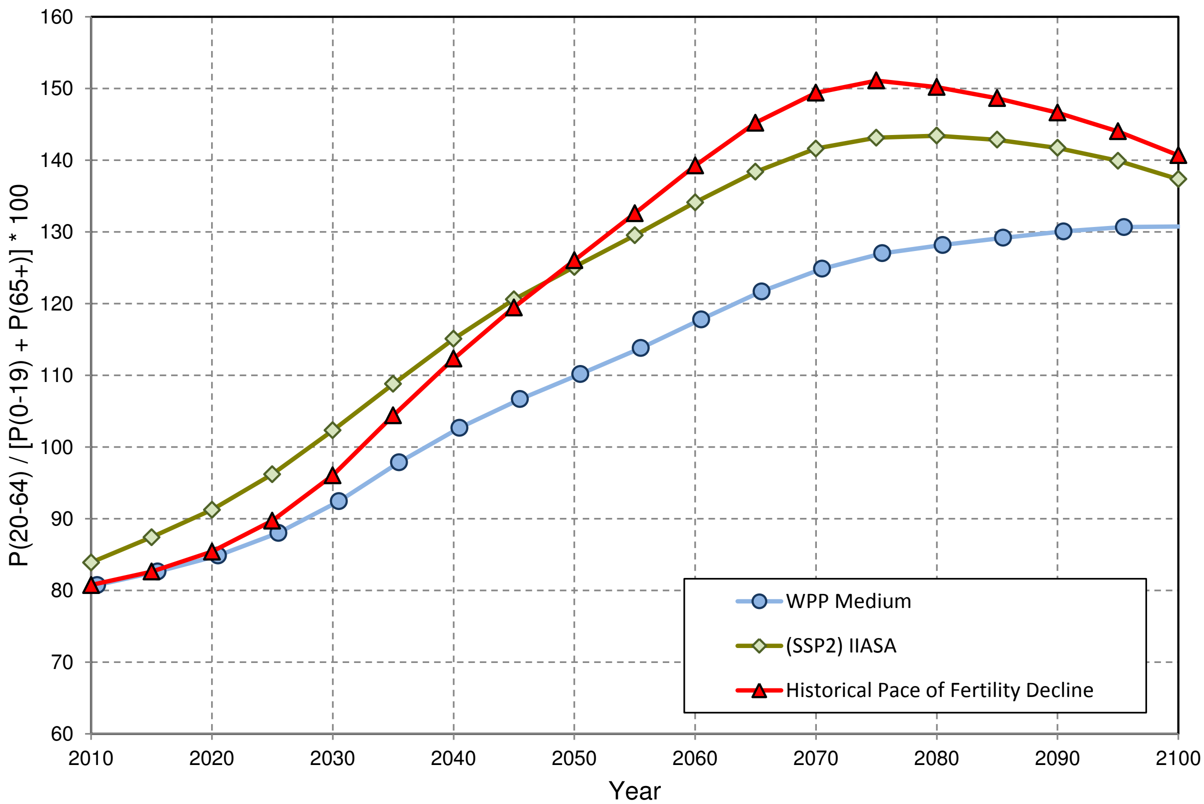
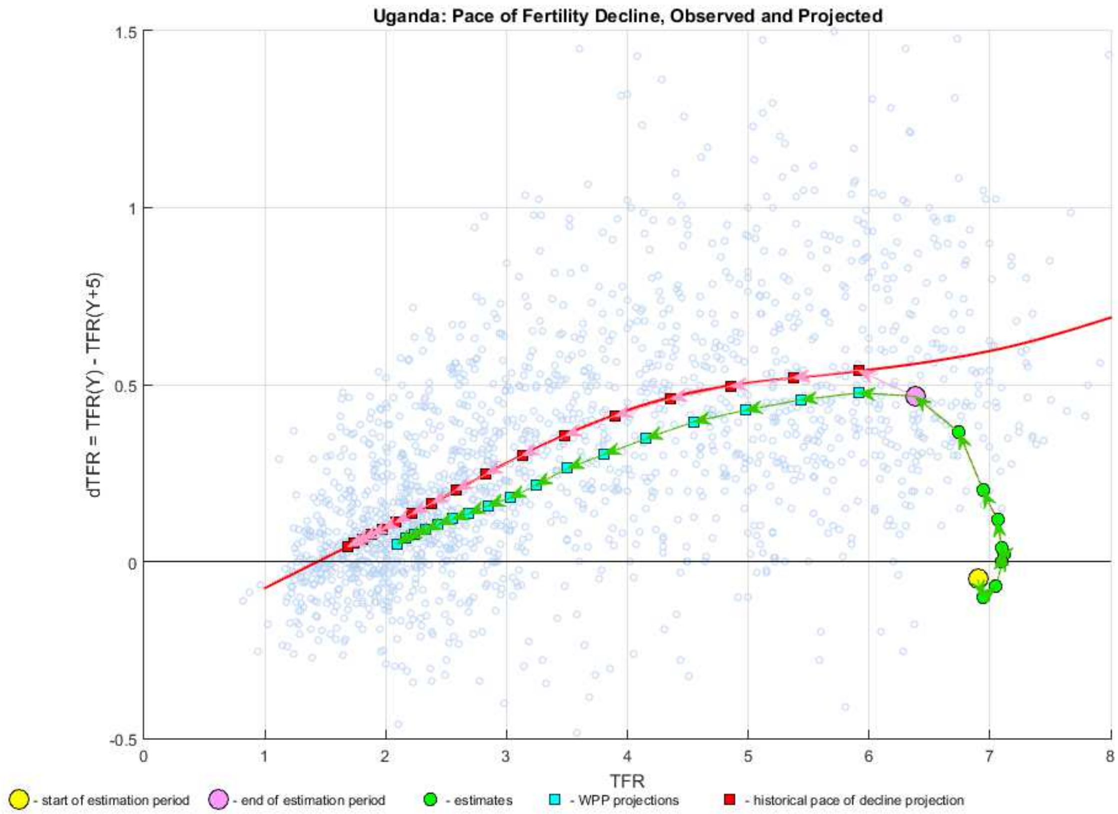
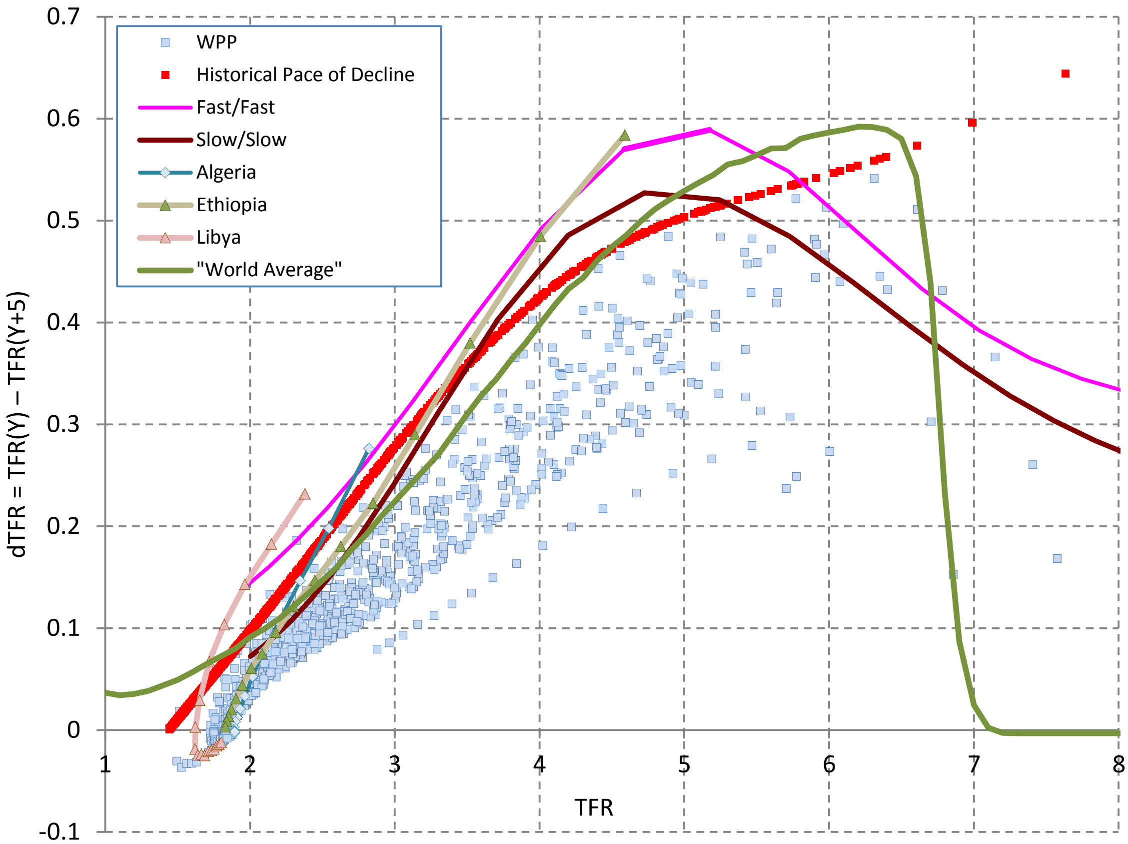


Figure 8 Estimated and Projected Pace of Fertility Decline in Uganda: UN projections and Projections Based on Historical Pace of Decline



The pale blue circles - empirical data on pace of fertility decline for fertility transition and post-transitional periods. The red curve – the average historical pace of fertility decline.

Figure 9 Projected Pace of Fertility Decline for all African Countries



"WPP" – projected pace of fertility decline for all African countries in the UN projections[11], the 2015 Revision of World Population Prospects.
"Historical Pace of Fertility Decline" – projected pace of fertility decline based on historical averages.
"Fast/Fast" & "Slow/Slow" – fertility decrements implied by "Fast/Fast" and "Slow/Slow" fertility models (United Nations, 2006).

"World Average" – World mean of the decline curve, Bayesian hierarchical model, Alkema et al. (2011), Fig. 4[12].

[11] An outlier, the only country with dTFR < 0.2 for TFR about 4, with atypically low fertility decrements is Zambia. See also http://kirillandreev.com/afrproj/fig01/894.png and http://www.un.org/en/development/desa/population/publications/dataset/fertility/wfd2015/894_tfr.png.

[12] Decrements for individual countries, series WPP, are from the 2015 Revision of World Population Prospects. The estimate of "World Average" was published based on empirical data from an earlier revision. As the model is unchanged and only data for a few additional years became available we are not expecting dramatic changes in the shape of "World Average" based on the data from 2015 Revision. Parameters of the BHM model are not polished by the UN Population Division.